ANALYSIS OF ACCELERATED FLOW OVER AN INSULATED WEDGE SURFACE USING VON KARMAN-POHLHAUSEN'S MOMENTUM INTEGRAL METHOD

Fazlar Rahman
Department of Mechanical and Production Engineering (MPE)
Ahsanullah University of Science and Technology (AUST), Tejgaon Industrial Area, Bangladesh
Corresponding Email: Fazlar19@hotmail.com

Abstract: *An accelerated flow over an insulated wedge surface is investigated for wedge angle in between 0.50 degree to 175 degree by using Von Karman-Pohlhausen's momentum integral method. The wedge surface is insulated at the leading edge and heating begin at the end of insulation zone. The effect of wedge angle on flow characteristics such as boundary layer thickness, momentum thickness, thermal boundary layer thickness and heat transfer coefficient are investigated. The equations of flow characteristics are derived for various wedge angles from the governing equations of Von Karman-Pohlhausen's momentum integral method and expressed in terms of Reynolds number, Prandtl number and Nusselt number. The results are plotted to investigate the flow within the boundary layer and found that separation of flow occurred earlier with increase of wedge angle to 105 degree and beyond. The results of flow characteristics for 0.5 degree wedge angle are compared with Blasius's exact solution of flat plate and also with VonKarman-Pohlhausen's solution of flat plate to validate the analysis presented in this paper. From the analysis, it is also revealed that Von Karman-Pohlhausen's momentum integral method is convenient to solve or draw outline of solution of flow over an arbitrary shaped object than Blasius's exact solution method.*

Keywords: *Flow over Wedge Surface; Boundary Layer Integral Method; Thermal Boundary Layer.*

INTRODUCTION

The mathematical complexity of convection heat transfer is related to the non-linearity of Navier-Stokes equation and coupling of fluid flow with thermal fields. The Navier-Stokes equation is elliptical in nature and quite difficult to get solution. The boundary layer concept was first introduced by Prandtl in 1904 by simplification of Navier-Stokes equation. The simplified Navier-Stock equation is known as Prandtl's boundary layer equations. It is easier to solved and exhibit completely different mathematical behavior than Navier-Stokes equation. The systematic calculation of Prandtl's boundary layer equations yields flow characteristics within the boundary layer of the fluid flow[1].

In 1908, Heinrich Blasius solved Prandtl's boundary layer equations into non-linear ordinary differential equations. Despite the importance of Blasius's solution, in 1921 Von Karman, obtained momentum-integral equation through the simple expedient of integrating the boundary-layer equations across the boundary layer. Later the Von Karman momentum integral equations are refined by Ernst Pohlhasuen, which is known as Von Karman-Pohlhasuen's momentum integral method. It is well suited to generate a quick outline of a solution in complex cases and arbitrary shaped objects. The Von Karman-Pohlhasuen's momentum integral methods provide significant mathematical simplification by reducing the number of independent variables. It gives an insight to the behavior of fluid particles within the boundary layer and easier to work with[1].

For mathematical simplification, Von Karman-Pohlhasuen's momentum integral method is extensively used to solve a wide range of problems in fluid flow, convection heat and mass transfer from complex to arbitrary shaped object, where Blasius's exact solution is difficult to achieved[2-3].

Flow over wedge surface is an important phenomenon in aerodynamics to design the aircraft's wing, missiles and other flying and floating objects, as well as, designing of other thermo-mechanical equipments and devices. There are numerous research papers published to investigate flow over edge surface with various conditions and situations. But only few of them investigated the flow over insulated wedge surface. Before mathematical formulation, the related papers and books are well studied[1-7]. Zhang, Yuwen[8] worked with flow over insulated web surface with a constant velocity U_∞ i.e. assuming, $\frac{dU}{dx} = 0$by using boundary layer integral methods. Basu, B[9] investigated flow over web surface without insulation with Blasius equations. Keshtkar, Mohammad[10] investigated flow over wedge surface with varying viscosity and heat generation of Falkner-Skan flow. Akcay, Mehmet[11] investigated flow over a moving wedge surface with wall mass injection of non-Newtonian fluid by using Prandtl's boundary layer hypothesis. Ashwini, G.[12]investigated unsteady MHD (Magneto-hydrodynamics or magneto fluid dynamics) decelerating flow over wedge surface with heat generation or absorption in presence of magnetic field by using boundary layer equations and Keller-box solution method. Padet, J.[13] investigated transient convective heat transfer over wedge surface with differential and integral method without insulation over the wedge surface. Butt, S., Adnan[14] investigated flow and heat transfer over a static and a moving wedge surface by similarity method without insulation. Seddeek, M.[15] investigated MHD Falkner-Skan flow and heat transfer over wedge surface with variable viscosity and thermal conductivity by similarity solution. Mukhopadhyay, Swati[16] investigated boundary layer flow and heat transfer of a Casson fluid over porous wedge surface. Before mathematical formulation, all papers[1-20] are well studied and found that Von Karman-Pohlhausen's integral formulation is easier to work with complex and arbitrary shaped object, as well as, the accuracy of the solution is acceptable within the engineering and research communities.

The flow over wedge surface is investigated by Von Karman-Pohlhausen's integral methods for various wedge angles in between $0.5^0<\alpha<175^0$.

METHODOLOGY

Considering two-dimensional laminar steady flow over a wedge surface, as shown in the **Fig.-1**. Where x is measured along parallel to the wedge surface and y is measured along perpendicular to the wedge surface. The surface of the wedge is insulated in between $0< x <x_0$. The free stream velocity and temperature are U_∞ and T_∞ respectively. The temperature of the wedge surface and velocity of fluid over wedge surface are T_w and$U(x) = c\,x^m$respectively, where $m = \frac{\beta}{2-\beta}$ (m>0),$\beta = \frac{\alpha}{180}$and α is the wedge angle in degree.

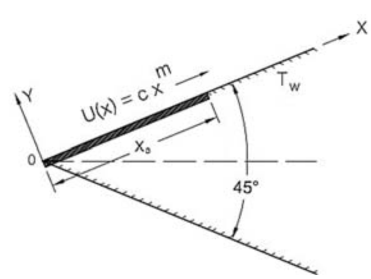

Fig.-1: Flow Over Wedge Surface.

Considering no generation of thermal energy, negligible dissipation, no gravity and no blowing or suction at the wedge surface. The wedge surface is insulated in between

$0<x<x_0$, so there is no thermal boundary layer developed at the edge and wall temperature of wedge surface is same as free stream fluid temperature within insulated region i.e.
$T_w = T_\infty$.

The thermal boundary layer will be developed at $x>x_0$ and $\delta_t(x)$ is less than $\delta(x)$. The ratio of $\frac{\delta_t(x)}{\delta(x)}$ will be less than 1 i.e. $\xi = \frac{\delta_t(x)}{\delta(x)}<1$.

GOVERNING EQUATIONS

All the governing equations of Von Karman-Pohlhasuen's momentum integral methods are readily available in H. Schlichting[2]. The velocity of fluid over the wedge surface is given by the equation,

$$U(x) = c\,x^m \quad (1)$$

For steady flow with zero incidence and laminar boundary layer, the momentum thickness $\delta_2(x)$ is given by the following integral equation,

$$\frac{\rho\, U(x)\delta_2{}^2(x)}{\mu} = \frac{0.47}{U(x)^5}\int_0^x U(x)^5\,dx \quad (2)$$

The relationship in between boundary layer shape factor $\Lambda(x)$ and boundary layer thickness $\delta(x)$ is given by the following equation,

$$\Lambda(x) = \frac{\rho\delta^2(x)}{\mu}\frac{dU(x)}{dx} \quad (3)$$

The dimensionless ratio of local velocity u to the free stream velocity U_∞ is given by the following equation,

$$\frac{u}{U_\infty} = (2\eta - 2\eta^3 + \eta^4) + \frac{\Lambda(x)}{6}(\eta - 3\eta^2 + 3\eta^3 - \eta^4) \quad (4)$$

Where $\eta = \frac{y}{\delta(x)}$

Flat plate with zero incidences, the ratio of momentum thickness $\delta_2(x)$ and boundary layer thickness $\delta(x)$ is given by the following equation,

$$\frac{\delta_2(x)}{\delta(x)} = \frac{37}{315} - \frac{\Lambda(x)}{945} - \frac{\Lambda(x)^2}{9072} \quad (5)$$

The momentum layer shape factor K(x) is related to the momentum thickness $\delta_2(x)$ by the following equation,

$$K(x) = \frac{\rho\delta_2{}^2(x)}{\mu}\frac{dU(x)}{dx} \quad (6)$$

$H(\xi)$ is the universal function of $\xi(x)$ and $\xi(x) = \frac{\delta_t(x}{\delta(x)}$. The value of function $H(\xi)$ is given by the following equations,

For $\xi \le 1$, $H(\xi) = \frac{2}{15}\xi - \frac{3}{140}\xi^3 + \frac{1}{180}\xi^4$ and for $\xi > 1$,

$$H(\xi) = \frac{3}{10} - \frac{3}{10}\xi^{-1} + \frac{2}{15}\xi^{-2} + \frac{3}{140}\xi^{-4} + \frac{1}{180}\xi^{-5} \quad (7a\text{-}b)$$

The differential form of integral energy equation for the thermal boundary layer $\delta_t(x)$ is given by the following equation,

$$\frac{d}{dx}[U(x)\,H(\xi)\delta_t(x)] = \frac{2\,k}{\rho C_p}\frac{1}{\delta_t(x)} \quad (8)$$

The local rate of heat transfer is given by the following equation,

$$q_x = \frac{2\,k}{\delta_t(x)}(T_w - T_\infty) \quad (9a)$$

In terms of heat flux or coefficient of heat transfer equation (9a) can be written as,

$$q_x = h_x(T_w - T_\infty) \quad (9b)$$

Where $h_x = \frac{2\,k}{\delta_t(x)}$ (9c)

MATHEMATICAL FORMULATION

The equations related to the flow characteristics over the wedge surface are derived systematically as shown below by using governing equations (1) to (9) and expressed each equation in terms of non-dimensional constants of Reynolds number, Prandtl's number and Nusselt number. The effect of flow characteristics for various wedge angle from $\alpha = 0.5^0$ to $\alpha = 175^0$ are investigated.

Momentum Thickness, $\delta_2(x)$:
From the governing equations (1) and (2),

$$\delta_2{}^2(x) = \frac{0.47\upsilon}{(c\,x^m)^6}\int_0^x (c\,x^m)^5\ dx \quad (10a)$$

After integrating and applying boundary condition at x = 0, $\delta_2(x) = 0$, equation (10a) becomes,

$$\delta_2{}^2(x) = \left(\frac{0.47\,\upsilon}{c}\right)\left(\frac{x^{-m+1}}{5m+1}\right) \quad (10b)$$

$$\delta_2(x) = \left\{\frac{0.47\,\upsilon}{c\,(5m+1)}\right\}^{\frac{1}{2}}(x)^{\frac{-m+1}{2}} \quad (10c)$$

Local Reynolds number,

$$R_{ex} = \frac{U(x).\,x}{\nu} = \frac{c\,x^{m+1}}{\nu}$$

Substituting expression of local Reynolds numberR_{ex}in equation (10c) and rearranging variables,

$$\frac{\delta_2(x)}{x} = \left(\frac{0.47}{5m+1}\right)^{\frac{1}{2}}\frac{1}{\sqrt{R_{ex}}} \quad (10d)$$

Boundary Layer Thickness, $\delta(x)$:
From the governing equation (1) and (3),

$$\Lambda(x) = \frac{\delta^2(x)}{\nu}(c\,m\,x^{m-1}) \quad (11a)$$

$$\delta^2(x) = \left(\frac{\nu}{c\,m}\right)(x^{-m+1})\Lambda(x) \quad (11b)$$

$$\delta(x) = \left(\frac{\nu}{c\,m}\right)^{\frac{1}{2}}\left(x^{\frac{-m+1}{2}}\right)\Lambda(x)^{\frac{1}{2}} \quad (11c)$$

From the equations (10c), (11c) and governing equation (5),

$$\frac{1}{63}\left\{\frac{37}{5} - \frac{\Lambda(x)}{15} - \frac{\Lambda(x)^2}{144}\right\} = \left(\frac{0.47m}{5m+1}\right)^{\frac{1}{2}}\left\{\frac{1}{\Lambda(x)}\right\}^{\frac{1}{2}} \quad (12a)$$

$$\frac{\Lambda(x)^{\frac{5}{2}}}{144} + \frac{\Lambda(x)^{\frac{3}{2}}}{15} - \frac{37\Lambda(x)^{\frac{1}{2}}}{5} + 63\left(\frac{0.47m}{5m+1}\right)^{\frac{1}{2}} = 0 \quad (12b)$$

$$\frac{\Lambda(x)^{\frac{5}{2}}}{144} + \frac{\Lambda(x)^{\frac{3}{2}}}{15} - \frac{37\Lambda(x)^{\frac{1}{2}}}{5} + C_2 = 0 \quad (12c)$$

$$\text{Where}C_2 = 63\left(\frac{0.47m}{5m+1}\right)^{\frac{1}{2}} \quad (12d)$$

The value of K(x) is found from the governing equation (1), (6) and equation (10c).

$$K(x) = \frac{1}{\nu}\left(\frac{0.47\,\upsilon}{c}\right)\left(\frac{x^{-m+1}}{5m+1}\right)(c\,m\,x^{m-1}) \quad (13a)$$

$$K(x) = \frac{0.47m}{5m+1} \quad (13b)$$

Expressing equation (11c) in terms of local Reynolds numberR_{ex},

$$\frac{\delta(x)}{x} = \left\{\frac{\Lambda(x)}{m}\right\}^{\frac{1}{2}}\frac{1}{\sqrt{R_{ex}}} \quad (14a)$$

$$\delta(x) = \left\{\frac{\nu\Lambda(x)}{c\,m}\right\}^{\frac{1}{2}}\left(x^{\frac{-m+1}{2}}\right) \quad (14b)$$

$$\delta(x) = C_3\left(x^{\frac{-m+1}{2}}\right) \quad (14c)$$

$$\text{Where}C_3 = \left\{\frac{\nu\Lambda(x)}{c\,m}\right\}^{\frac{1}{2}} \quad (14d)$$

Thermal Boundary Layer Thickness, $\delta_t(x)$:
The thermal boundary layer $\delta_t(x)$ will be developed at $x \geq x_0$ and boundary layer will be developed at x=0 and $\xi(x) = \frac{\delta_t(x)}{\delta(x)}$ will be less than 1 ($\xi < 1$).

From governing equation (7a),

$$H(\xi) = \frac{2}{15}\left\{\frac{\delta_t(x)}{\delta(x)}\right\} - \frac{3}{140}\left\{\frac{\delta_t(x)}{\delta(x)}\right\}^3 + \frac{1}{180}\left\{\frac{\delta_t(x)}{\delta(x)}\right\}^4 \quad (15a)$$

Neglecting higher order terms and substituting the value of $\delta(x)$ from the equation (14b),

$$H(\xi) = \frac{2}{15}\left\{\frac{c\,m}{\nu\Lambda(x)}\right\}^{\frac{1}{2}}\left(x^{\frac{m-1}{2}}\right)\delta_t(x) \qquad (15b)$$

Substituting value of $H(\xi)$ from equation (15b) and $U(x)$ from governing equation (1) to the governing equation (8),

$$\frac{d}{dx}\left[c x^m \frac{2}{15}\left\{\frac{c\,m}{\nu\Lambda(x)}\right\}^{\frac{1}{2}}\left(x^{\frac{m-1}{2}}\right)\delta_t{}^2(x)\right] = \left(\frac{2\,k}{\rho C_p}\right)\frac{1}{\delta_t(x)} \qquad (16a)$$

Rearranging the variables,

$$\left\{x^{\frac{3m-1}{2}}\delta_t{}^2(x)\right\}^{\frac{1}{2}} d\left\{x^{\frac{3m-1}{2}}\delta_t{}^2(x)\right\} = \frac{15}{2}\left(\frac{\nu}{c}\right)^{\frac{3}{2}}\left\{\frac{\Lambda(x)}{m}\right\}^{\frac{1}{2}}\left(\frac{k}{\mu C_p}\right)\left(x^{\frac{3m-1}{4}}\right)dx \qquad (16b)$$

$$\frac{3}{2}\left\{x^{\frac{3m-1}{2}}\delta_t{}^2(x)\right\}^{\frac{3}{2}} = \left(\frac{\nu}{c}\right)^{\frac{3}{2}}\left\{\frac{\Lambda(x)}{m}\right\}^{\frac{1}{2}}\left(\frac{1}{P_r}\right)\left\{\frac{4}{3(m+1)}\right\}\left\{x^{\frac{3(m+1)}{4}}\right\} + A_1 \qquad (16c)$$

$$\left\{x^{\frac{3m-1}{2}}\delta_t{}^2(x)\right\}^{\frac{3}{2}} = 30\left(\frac{\nu}{c}\right)^{\frac{3}{2}}\left\{\frac{\Lambda(x)}{m}\right\}^{\frac{1}{2}}\left(\frac{1}{P_r}\right)\left(\frac{1}{m+1}\right)\left\{x^{\frac{3(m+1)}{4}}\right\} + A_2 \qquad (16d)$$

Where A_1 and A_2 are integration constants. Applying boundary condition in the equation (16d), at $x = x_0$, $\delta_t(x) = 0$.

$$A_2 = -30\left(\frac{\nu}{c}\right)^{\frac{3}{2}}\left\{\frac{\Lambda(x)}{m}\right\}^{\frac{1}{2}}\left(\frac{1}{P_r}\right)\left(\frac{1}{m+1}\right)\left\{x_0^{\frac{3(m+1)}{4}}\right\}$$

Substituting the value of constant A_2 in equation (16d),

$$\left[x^{\frac{3m-1}{2}}\delta_t{}^2(x)\right]^{\frac{3}{2}} = \left\{\frac{30}{P_r(m+1)}\right\}\left(\frac{\nu}{c}\right)^{\frac{3}{2}}\left\{\frac{\Lambda(x)}{m}\right\}^{\frac{1}{2}} x^{\frac{3(m+1)}{4}}\left[1-\left(\frac{x_0}{x}\right)^{\frac{3(m+1)}{4}}\right] \qquad (16e)$$

$$\delta_t{}^2(x) = \left\{\frac{30}{P_r(m+1)}\right\}^{\frac{2}{3}}\left(\frac{\nu}{c}\right)\left\{\frac{\Lambda(x)}{m}\right\}^{\frac{1}{3}} x^{-m+1}\left[1-\left(\frac{x_0}{x}\right)^{\frac{3(m+1)}{4}}\right]^{\frac{2}{3}} \qquad (16f)$$

$$\delta_t(`x) = \left\{\frac{30}{P_r(m+1)}\right\}^{\frac{1}{3}}\left\{\frac{\Lambda(x)}{m}\right\}^{\frac{1}{6}}\left(\frac{\nu}{c\,x^m\,x}\right)^{\frac{1}{2}} x\left[1-\left(\frac{x_0}{x}\right)^{\frac{3(m+1)}{4}}\right]^{\frac{1}{3}} \qquad (17a)$$

Rearranging the variables of equation (17a),

$$\delta_t(x) = C_4\left(\frac{1}{P_r}\right)^{\frac{1}{3}}\left(\frac{\nu}{c}\right)^{\frac{1}{2}} x^{\frac{-m+1}{2}}\left[1-\left(\frac{x_0}{x}\right)^{\frac{3(m+1)}{4}}\right]^{\frac{1}{3}} \qquad (17b)$$

Expressing equation (17b) terms of local Reynolds number R_{ex},

$$\delta_t(x) = \left(\frac{30}{m+1}\right)^{\frac{1}{3}}\left\{\frac{\Lambda(x)}{m}\right\}^{\frac{1}{6}}\left\{\frac{1}{(R_{ex})^{\frac{1}{2}}(P_r)^{\frac{1}{3}}}\right\} x\left[1-\left(\frac{x_0}{x}\right)^{\frac{3(m+1)}{4}}\right]^{\frac{1}{3}} \qquad (18a)$$

$$\delta_t(x) = C_4\frac{1}{R_{ex}^{\frac{1}{2}}P_r^{\frac{1}{3}}}\,x\left[1-\left(\frac{x_0}{x}\right)^{\frac{3(m+1)}{4}}\right]^{\frac{1}{3}} \qquad (18b)$$

$$\text{Where, } C_4 = \left(\frac{30}{m+1}\right)^{\frac{1}{3}}\left\{\frac{\Lambda(x)}{m}\right\}^{\frac{1}{6}} \qquad (18c)$$

The value of $\delta_t(x)$ can be found if value of x_0 or local Reynolds number R_{ex0} and also Prandtl's number are known. The value of x_0 can be found from the local Reynolds number R_{exo} at the end of insulation of the wedge surface, i.e. at $x = x_0$.

$$R_{ex0} = \frac{c{x_0}^{m+1}}{\nu} \qquad (19a)$$

$$x_0 = \left(\frac{R_{ex0}\nu}{c}\right)^{\frac{1}{m+1}} \quad (19b)$$

The values of shape factor $\Lambda(x)$ are found by solving equation (12c) numerically after substituting value of C_2 from equation (12d) and value of $K(x)$ is found from equation (13b). The value of C_3 and C_4 are found from the equations (14d) and (18c) respectively. The value of $K(x)$ and $\Lambda(x)$ for different wedge angle factors β, are shown in the **Table-1**. The results of **Table-1** are found analogous to the Holstein and Bohlen solution[2] for $\Lambda(x)$ and $K(x)$. It is assumed that the value of flow constant c =50 i.e. $U(x) = 50\,x^m$.

Table-1: Value of $\Lambda(x)$, $K(x)$, C_2, C_3 & C_4

α^0	β	m	$\Lambda(x)$	$K(x)$	C_2	C_3	C_4
0.5	1/360	1/719	0.0471	0.0007	1.605	0.00320	5.586
15	1/12	1/23	1.2479	0.0168	8.162	0.00295	5.360
30	1/6	1/11	2.2390	0.0294	10.798	0.00273	5.149
45	1/4	1/7	3.0566	0.0392	12.468	0.00254	4.952
60	1/3	1/5	3.7507	0.0470	13.658	0.00238	4.766
75	5/12	5/19	4.3532	0.0534	14.560	0.00224	4.588
90	1/2	1/3	4.8853	0.0588	15.270	0.00210	4.416
105	7/12	7/17	5.2098	0.0633	15.847	0.00196	4.228
120	2/3	1/2	5.7969	0.0671	16.325	0.00187	4.084
130	13/18	13/23	6.0660	0.0694	16.600	0.00180	3.975
145	29/36	29/43	6.4447	0.0725	16.963	0.00170	3.812
160	8/9	4/5	6.7981	0.0752	17.276	0.00160	3.649
175	35/36	35/37	7.1306	0.0776	17.549	0.00151	3.485

Heat Transfer Coefficient, h_x:

Substituting the value of $\delta_t(x)$ from equation (18b) to the governing equation (9c),

$$h_x = \frac{2k}{x}\left(\frac{R_{ex}^{\frac{1}{2}}P_r^{\frac{1}{3}}}{C_4}\right)\left[1-\left(\frac{x_0}{x}\right)^{\frac{3(m+1)}{4}}\right]^{-\frac{1}{3}} \quad (20a)$$

Substituting value of C_4 in equation (20a),

$$h_x = \frac{2k}{x}\left(\frac{m+1}{30}\right)^{\frac{1}{3}}\left\{\frac{m}{\Lambda(x)}\right\}^{\frac{1}{6}}\left(R_{ex}^{\frac{1}{2}}P_r^{\frac{1}{3}}\right)\left[1-\left(\frac{x_0}{x}\right)^{\frac{3(m+1)}{4}}\right]^{-\frac{1}{3}} \quad (20b)$$

Expanding local Reynolds number R_{ex},

$$h_x = \frac{2k}{C_4}\left(\frac{c}{\upsilon}\right)^{\frac{1}{2}}P_r^{\frac{1}{3}}\left(x^{\frac{m-1}{2}}\right)\left[1-\left(\frac{x_0}{x}\right)^{\frac{3(m+1)}{4}}\right]^{-\frac{1}{3}} \quad (20c)$$

Expressing in terms of local Reynolds R_{ex} and Nusselts number N_{ux} equation (20c) yields,

$$N_{ux} = 2\left(\frac{m+1}{30}\right)^{\frac{1}{3}}\left\{\frac{m}{\Lambda(x)}\right\}^{\frac{1}{6}}\left(R_{ex}^{\frac{1}{2}}P_r^{\frac{1}{3}}\right)\left[1-\left(\frac{x_0}{x}\right)^{\frac{3(m+1)}{4}}\right]^{-\frac{1}{3}} \quad (21)$$

RESULTS AND DISCUSSION

Considering, air is flowing over the wedge surface at 20 ^{0}C i.e. T_∞ = 20 ^{0}C. Properties of air at 20^{0}C are, Density of air, $\rho = 1.2\ \frac{Kg}{m^3}$

Specific heat at constant pressure, C_p=1.005 $\frac{KJ}{Kg\,K}$

Viscosity of air, μ=1.82 x $10^{-5}\frac{Kg}{m\,S}$

Kinematic viscosity, ν =15.2 x $10^{-6}\frac{m^2}{S}$

Thermal conductivity, K = 0.0257 $\frac{W}{m\,K}$

Prandtl number, P_r = 0.712

Boundary Layer Thickness δ(x):

The boundary layer thickness, δ(x) is plotted in **Fig.-2(a)** and **Fig.-2(b)** from the equation (14b) for various wedge angle factors β and velocity constant c =50 or local velocity over the wedge surface, $U(x) = 50\,x^m$.

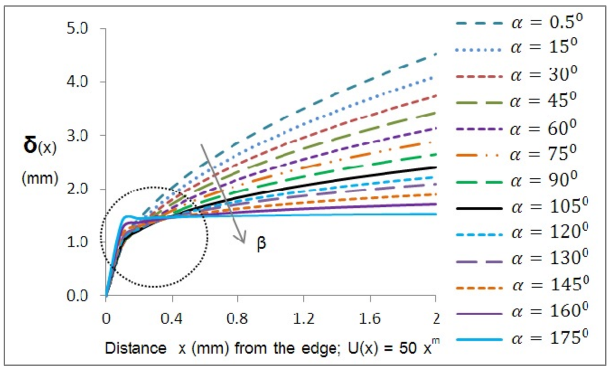

Fig.-2(a): Boundary Layer Thickness δ(x).

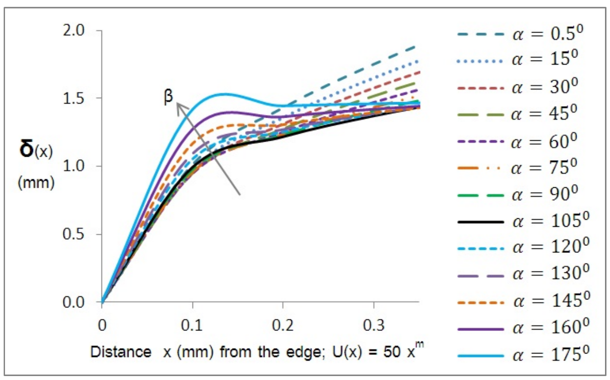

Fig.-2(b): Close View of Fig.-2(a).

From **Fig.-2(a)** and **2(b)**, it is observed that boundary layer thickness δ(x) increased with increasing of wedge angle factor β and flow separated earlier with increasing of β, which makes agreement with the real situation. Flow is separated quickly with increasing of wedge angle to 105^0 and beyond. Substituting the value of Λ(x) =0.0471 and m = 1/719 for wedge angle α = 0.5^0 from **Table-1** to the equation (14a), the equation of boundary layer thickness δ(x) becomes,

$$\frac{\delta(x)}{x} = \frac{5.82}{\sqrt{R_{ex}}} \quad (22)$$

Solution of boundary layer thickness δ(x) of flat plate with zero incidences by Von Karman-Pohlhausen's momentum integral method and Blasius's exact solution[2] are given by the following equations respectively,

$$\frac{\delta(x)}{x} = \frac{5.84}{\sqrt{R_{ex}}} \quad (23)$$

$$\frac{\delta(x)}{x} = \frac{5.0}{\sqrt{R_{ex}}} \quad (24)$$

The equation (22) makes good agreement with equation (23) and also makes satisfactory agreement with equation (24).

Momentum Thickness $\delta_2(x)$:

The momentum thickness $\delta_2(x)$ is plotted in **Fig.-3(a)** and **Fig-3(b)** from the equation (10c) for various wedge angle factors β and velocity constant c =50 or local velocity over the wedge surface $U(x) = 50\,x^m$.

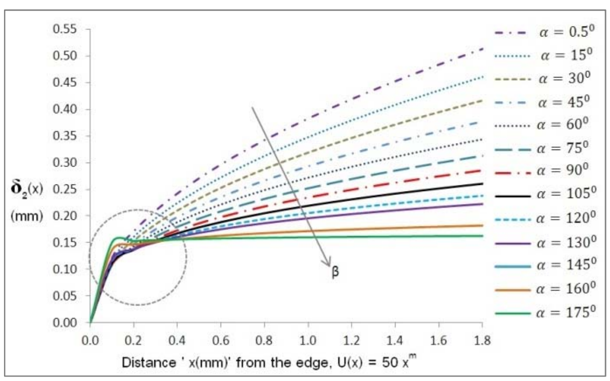

Fig.-3(a): Momentum Thickness δ2(x).

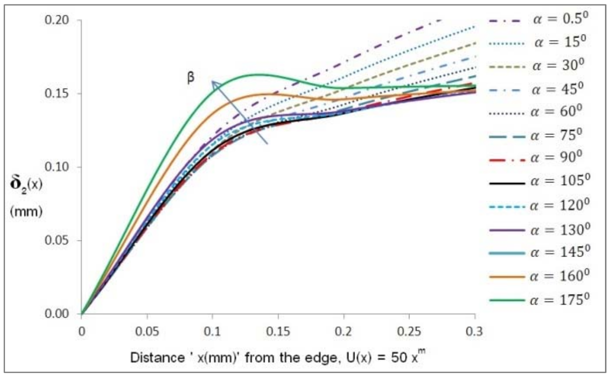

Fig.-3(b): Close View Fig.-3(a).

Form **Fig.-3(a) and 3(b)**, it is observed that the momentum thickness $\delta_2(x)$ is analogous to the boundary layer thickness $\delta(x)$ for various wedge angle factors β; which validates the acceptance of accuracy of the solution of momentum thickness $\delta_2(x)$. Substituting the value of m for wedge angle $\alpha = 0.5^0$ from **Table-1** to the equation (10d), the momentum thickness $\delta_2(x)$ becomes,

$$\frac{\delta_2(x)}{x} = \frac{0.683}{\sqrt{R_{ex}}} \tag{25}$$

Solution of momentum thickness $\delta_2(x)$ for flat plate with zero incidence by Von Karman-Pohlhasuen's momentum integral method and Blasius's exact solution are given by the following equations respectively,

$$\frac{\delta_2(x)}{x} = \frac{0.686}{\sqrt{R_{ex}}} \tag{26}$$

$$\frac{\delta_2(x)}{x} = \frac{0.664}{\sqrt{R_{ex}}} \tag{27}$$

The equation (25) makes very good agreement with equation (26) and also satisfactory agreement with equation (27).

Velocity Distribution in Laminar Boundary Layer:

The velocity distribution in the laminar boundary layer is found by solving governing equation (4). Substituting value of $\Lambda(x)$ for different wedge angles from **Table-1** to the governing equation (4) and numerically solve equation (4) for different values of $\eta = \frac{y}{\delta(x)}$ and $\frac{u}{U_\infty}$ respectively. The velocity distribution for laminar flow over wedge surface is shown in the **Fig.-4(a)** and **Fig.-4(b)**.

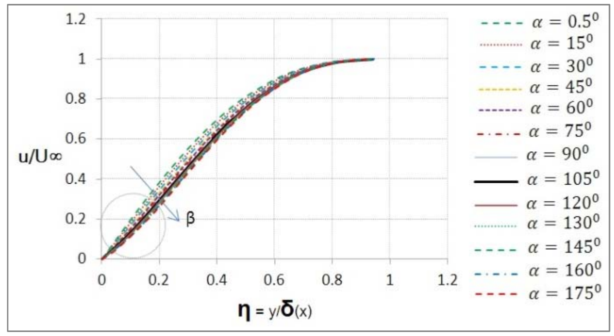

Fig.-4(a): Velocity Distribution for $U(x) = 50x^m$.

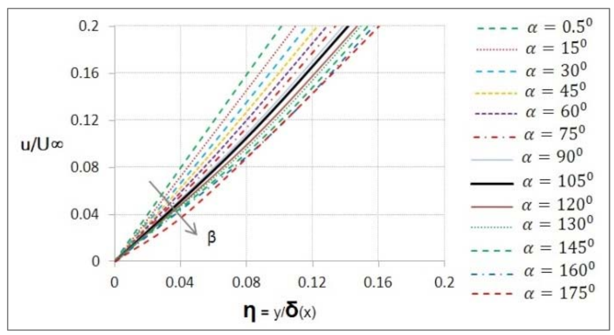

Fig.-4(b): Close View of Fig.-4(a).

From **Fig.-4(a)** and **4(b)**, it is observed that the boundary layer separated earlier with increasing of wedge angle to 105^0 and beyond. The profiles of velocity distribution over wedge surface for different wedge angles are found realistic and it makes good agreement with the velocity distribution of Blasius's exact solution for flow over wedge surface for laminar flow[2].

Thermal Boundary Thickness $\delta_t(x)$:

If the value of local Reynolds number, R_{ex0} at the end of the insulation is known then value of x_0 can be found from equation (19b).

The thermal boundary thickness $\delta_t(x)$ is plotted in **Fig.-5** by substituting the value of x_0 for local Reynolds number R_{ex0}= 1000, Prandtl number P_r = 0.712 and value of C_4 from **Table-1** to the equation (17b).

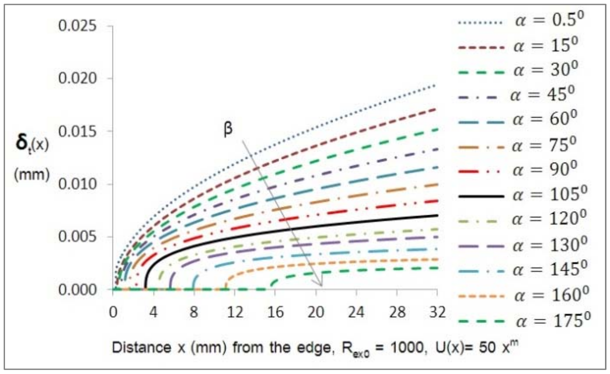

Fig.-5: Thermal Boundary Layer Thickness $\delta_t(x)$ for R_{ex0} = 1000; P_r = 0.712

If there is no insulation in the wedge surface i.e. x_0 = 0, the equation (18b) yields,

$$\delta_t(x) = C_4\left(\frac{1}{R_{ex}^{\frac{1}{2}}P_r^{\frac{1}{3}}}\right)x \qquad (28)$$

Substituting value of C_4 for wedge angle α =0.5^0 from the **Table-1** to the equation (28),

$$\delta_t(x) = \frac{5.586\,x}{{P_r}^{1/3}\sqrt{R_{ex}}} \qquad (29)$$

Exact solution of thermal boundary layer for flow over flat plate with zero incidences is given by the following equation[2],

$$\delta_t(x) = \frac{5.0\,x}{{P_r}^{1/3}\sqrt{R_{ex}}} \qquad (30)$$

The equation (29) makes satisfactory agreement with equation (30).

Heat Transfer Coefficient, h_x:

The heat transfer coefficient h_x is plotted in **Fig.-6(a)** and **Fig.-6(b)** by substituting value of x_0 for local Reynolds number R_{exo} = 1000, Prandtl number P_r = 0.712 and value of C_4 from **Table-1** to the equation (20c).

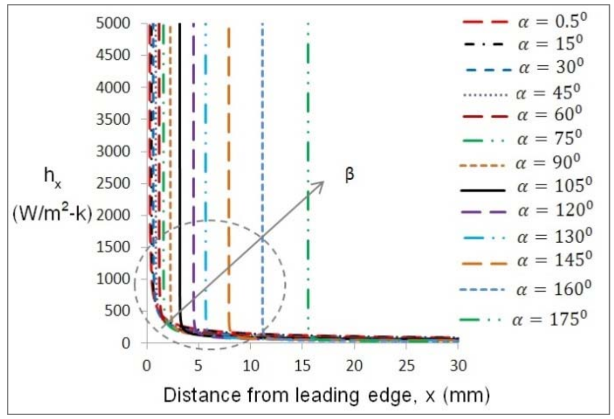

Fig.-6(a): Heat Transfer Coefficient, h_x for R_{ex0} = 1000, Pr = 0.712 for U(x) = 50 x^m.

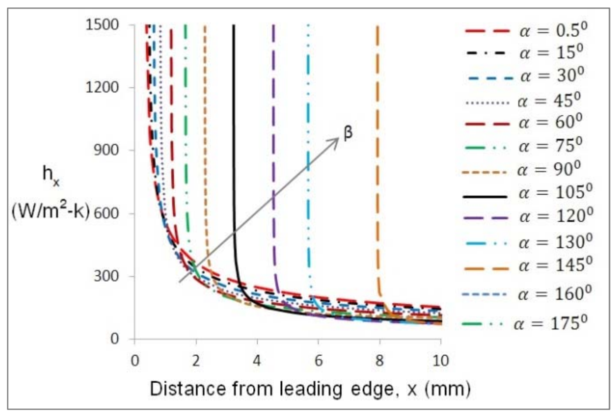

Fig.-6(b): Close View of Fig.-6(a)

Substituting value C_4 and Λ(x) for α = 0.5^0 from the **Table-1** to equation (21),

$$N_{ux} = 0.358\left(R_{ex}^{\frac{1}{2}}P_r^{\frac{1}{3}}\right)\left[1-\left(\frac{x_0}{x}\right)^{\frac{3}{4}}\right]^{-\frac{1}{3}} \qquad (31)$$

If there is no insulation on the web surface, i.e. x_0 = 0 then equation (31) becomes,

$$N_{ux} = 0.358\left(R_{ex}^{\frac{1}{2}}P_r^{\frac{1}{3}}\right) \qquad (32)$$

The numerical solution of heat transfer for flow over flat plate[2] is given by,

$$N_{ux} = 0.332\left(R_{ex}^{\frac{1}{2}} P_r^{\frac{1}{3}}\right) \quad (33)$$

The equations (31) and (32) make very good agreement with Zhang, Yuwen (2010)[8] and also with numerical solution of heat transfer over flat plate equation (33).

The variations of solutions of boundary layer thickness δ(x), momentum layer thickness $\delta_2(x)$, thermal boundary layer thickness $\delta_t(x)$ and heat transfer coefficient h_x from the Blasius's exact solutions or numerical solutions are within acceptable limit. Because solutions presented in this paper for 0.5^0 angle wedge surface are compared with Blasius's exact solutions for flat plate.

Boundary Layer Thickness δ(x), Thermal Boundary Layer Thickness $\delta_t(x)$ and Heat Transfer Coefficient h_x:

The relative characteristics of heat transfer coefficient h_x, boundary layer thickness δ(x) and thermal boundary layer thickness $\delta_t(x)$ for wedge angles $\alpha = 0.5^0$, $\alpha = 30^0$, $\alpha=75^0$ and $\alpha=120^0$ are shown in the **Fig.-7(a)** and **Fig.-7(b)**.

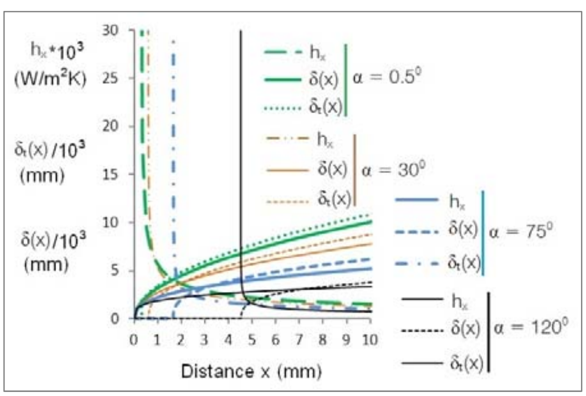

Fig.-7(a): Heat Transfer Coefficient, h_x; Boundary Layer Thickness δ(x) and Thermal Boundary Layer Thickness $\delta_t(x)$.

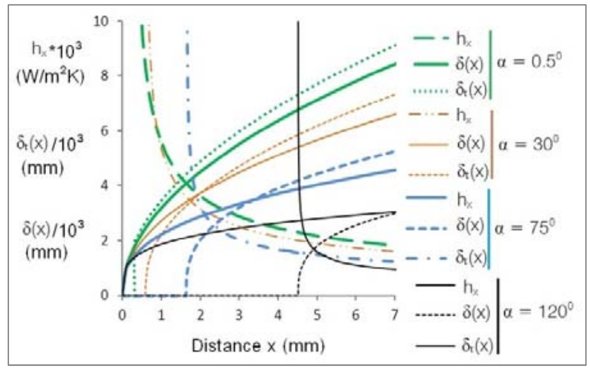

Fig.-7(b): Close View of Fig.-7(a).

From **Fig.-7(a)** and **7(b)**, it is observed that the boundary layer thickness δ(x) and thermal boundary layer thickness $\delta_t(x)$ increases gradually with decreasing of heat transfer coefficient h_x or with increasing of length of insulation.

CONCLUSION

From the analysis, presented in this paper the following conclusions are made:

(1). The flow characteristics over wedge surface for wedge angle,$\alpha=0.5^0$ are found consistence with Von Karman-Pohlhausen's integral methods for flat plate, as well as, with Blasius's exact solution for flat plate.

(2). The boundary layer thickness and momentum thickness increase with increasing of wedge angle and boundary layer separates earlier with increasing of wedge angle to 105^0 and beyond.

(3). The thermal boundary layer thickness and heat transfer coefficient decrease with increasing of length of insulation over wedge surface. In other words, length of insulation should be increased with increasing of wedge angle to keep the same amount of heat flux.

(4). Von Karman-Pohlhausen's momentum integral method is the most simple and robust tool, which could be used to solve flow around complex or any arbitrary shaped object, as well as, it could be used for validation of test data and other solution methods.

ACKNOWLEDGMENT

The author would like to acknowledge Prof. Dr. A. Haji-Sheikh, UTA, Arlington, TX for his valuable suggestions and guidelines to complete this research work.

NOMENCLATURE

C_p	Specific heat of fluid at constant pressure
$\delta(x)$	Boundary layer thickness is function x
$\delta_2(x)$	Momentum thickness function of x
$\delta_t(x)$	Thermal boundary thickness function of x
$H(\xi)$	Universal function of ξ
U_∞	Free stream velocity
$U(x)$	Velocity of over wedge, $U(x) = c\,x^m$
c	Velocity constant
m	Exponent of velocity
$\Lambda(x)$	Shape factor and function of $\delta(x)$ and x
$K(x)$	Shape factor and function of $\delta_2(x)$ and x
q_w	Heat transfer through wall
h_x	Heat flux or coefficient of heat transfer
P_r	Prandtl number, $P_r = \frac{\mu C_p}{K}$
R_{ex_0}	Reynolds number at distance x_0
R_{ex}	Local Reynolds number at distance x
T_w	Temperature of the wedge surface
T_∞	Temperature of free stream fluid
k	Thermal conductivity
N_{ux}	Nusselt Number, $N_{ux} = \frac{h\,x}{k}$
$\frac{u}{U_\infty}$	Dimensionless ratio of velocity

GREEK SYMBOLS

μ	Absolute viscosity of fluid
ρ	Density of the fluid
ν	Kinematic viscosity of fluid $\left(\nu = \frac{\mu}{\rho}\right)$
β	Wedge angle factor
α	Wedge angle in degree
η	Dimensionless ratio, $\frac{y}{\delta(x)}$
ξ	$\frac{\delta_t(x)}{\delta(x)}$ ($\xi < 1$ for this particular situation)

REFERENCES

[1] Anderson Jr., D. John, 2005, "Ludwig Prandtl's Boundary Layer", American Institute of Physics.

[2] Schlichting, Dr. Hermann, 1979, "Boundary Layer Theory", McGraw Hill.

[3] Schliching, H., 1949, "Lecture Series—Boundary Layer Theory- Part I — Laminar flows", NACA.

[4] Kackac, S. Sadik, 1994, "Convective Heat Transfer", CRC Press.

[5] Eckert, E. R., and Drake, 1986, "Analysis of Heat and Mass Transfer", CRC Press.

[6] Jiji, Latif M, 2006, "Heat Convection", Springer.

[7] Helmy, K., A, 2001, "An Integral Method for the Solution of the Boundary Layer Equation for Power-Law MHD Fluid", Indian Journal of Pure Applied Math.

[8] Zhang, Yuwen, 2010, "Integral solution of boundary layer equation", Thermal fluid central (www.thermalfluidscentral .org).

[9] Basu, B., 2011, "Chaotic behavior in the flow along a wedge modeled by the Blasius equation", Nonlin. Processes Geophys.

[10] Keshtkar, Mohammad, 2014, "The Falkner-Skan Flow over a Wedge with Variable Parameters", American-Eurasian Network for Scientific Information (AENSI).

[11] Akcay, Mehmet, 2009, "Flow of power-law fluids over a moving wedge surface with wall mass injection", Springer-Verlag.

[12] Ashwini, G., 2015, "Unsteady MHD Deceleration Flow over Wedge with Heat Generation/Absorption", American Institute of Science.

[13] Padet, J, 2004, "Transient Convective Heat Transfer", Presented at 10th Brazilian

Congress of Thermal Sciences and Engineering.

[14] Butt, S., Adnan; Ali, Asif, 2013, "Thermodynamical Analysis of the Flow and Heat Transfer over a Static and a Moving Wedge", Hindawi Publishing Corporation.

[15] M. A. Seddeek, 2009, "Similarity Solutions for a Steady MHD Falkner-Skan Flow and Heat Transfer Over a Wedge Considering the effects of variable Viscosity and Thermal Conductivity", Applications and Applied Mechanics.

[16] Mukhopadhyay, Swati; Mandal, C, Iswar; 2014, "Boundary layer flow and heat transfer of a Casson fluid past a symmetric porous wedge with surface heat flux", Chinese Physical Society and IOP Publishing Ltd.

[17] Kousar, Nabeela; Liao, Shijun; 2010, "Series Solution of Non-similarity Boundary-Layer Flows Over a Porous Wedge", Springer Science and Business Media.

[18] Mutlag, A., A.; Uddin, J., Md.; 2013; "Heat Transfer Analysis for Falkner-Skan Boundary Layer Flow Past a Stationary Wedge with Slips Boundary Conditions Considering Temperature-dependent Thermal Conductivity", SainsMalaysiana.

[19] Bararnia, H.; 2012; "Flow analysis for the Falkner–Skan Wedge Flow"; Current Science.

[20] Weybume, W., David; 2010; "A Realistic Theoretical Model for Laminar Flow over a Flat Plate"; Air Force Research Laboratory, MA.